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ANALYTICAL SOLUTION OF A FLOW PROBLEM IN THE NEIGHBOURHOOD OF THE BOUNDARY LAYER SEPARATION POINT ON A MOVING SURFACE*

V.V. SYCHEV

An accurate solution to a previously formulated boundary value problem /1/ for the boundary layer (BL) equations describing flow in the neighbourhood of the separation point on a moving surface is obtained.

The plane stationary flow of a viscous incompressible liquid in the neighbourhood of a release point on a surface which is moving downstream at a constant velocity is examined. As a result of the asymptotic analysis of the Navier-Stokes equations with large Reynolds numbers (R) it has been established /2/ that in the neighbourhood of the release point there is a region of interaction between the BL and the outer potential flow where a large unfavourable selfinduced pressure gradient is acting (the longitudinal and transverse dimensions of this region are quantities of the order of $R^{-1/3}$, see Fig.1). Upstream of this region, the flow is described by the BL equations; the pressure distribution outside this region is given (locally) by the solution of the theory of potential flows of an ideal liquid with free streamlines. The selfinduced pressure gradient leads to intense deceleration of the liquid inside the BL but does not cause flow separation, i.e. the appearance of a return flow in the interaction region /3/.

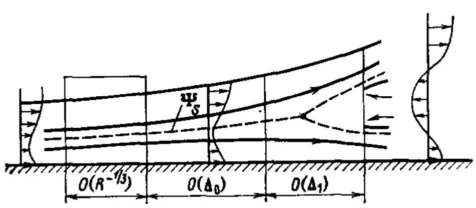

Fig.1

Subsequent analysis showed /1/ that the separation point must lie in a region situated inside the BL, at a short distance upstream of the interaction region. The asymptotic presentation of the solution of the Navier-Stokes equations (as $R \to \infty$) for this region take the form

$$x = L\Delta_1 x', \quad y = LR^{-1/2} y' \tag{1}$$
$$\psi = U_{\infty} L R^{-1/2} [\Psi_s + \Delta_1 \psi_0 (x', y') + \ldots]$$
$$p = p_{\infty} + \rho U_{\infty}^2 [\Delta_1^2 p_0 (x') + \ldots]$$
$$\Delta_1 = \sigma^{1/2} R^{-1/12}, \quad \sigma (\ln \sigma^{-1})^{2/3} = R^{-1/3}, \quad R = LU_{\infty}/\nu$$

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Here Ox and Oy are axes of a Cartesian rectangular system of coordinates directed parallel to the moving surface of the solid and perpendicular to it, respectively; the origin of the system of coordinates is placed in the region under investigation; ψ, p, ρ, are functions of the flow, pressure and density; Ψ_s is the streamline on which the separation point lies; U_{00} and p_{00} are the velocity and pressure on a free streamline; L is the characteristic dimension of the solid.

The required function $\psi_0(x', y')$ in (1), as usual, satisfies the Prandtl BL equation with a previously unknown pressure gradient $p'_0(x_0)$ and the conditions for matching with the solutions in neighbouring regions serve as the boundary conditions. Using the transformations

$$\begin{aligned} x' &= \gamma_0 \ln \alpha_0 + \gamma_0 X \\ y' &= a_0^{-1} Y + a_0^{-1} X + (3a_0)^{-1} \ln [3k^2/(4a_0)] \\ \psi_0 &= a_0\gamma_0\Psi, \quad p_0 = a_0^4\gamma_0^2 P \\ \gamma_0 &= [(^3/_4)\, k^2 a_0^2]^{-1/_3}, \quad \alpha_0 = (k/2)\,(3a_0^{-3})^{1/_2} \end{aligned} \tag{2}$$

the boundary value problem for flow in the region under investigation becomes

$$\begin{aligned} &\frac{\partial\Psi}{\partial Y}\frac{\partial^2\Psi}{\partial X\,\partial Y} - \frac{\partial\Psi}{\partial X}\frac{\partial^2\Psi}{\partial Y^2} + \frac{dP}{dX} = \frac{\partial^3\Psi}{\partial Y^3} \\ &\Psi = \frac{\partial^2\Psi}{\partial Y^2} = 0 \quad (Y = 0), \quad \Psi \to \exp(Y - X) \quad (Y \to \infty) \\ &\Psi \to 2\exp(-X)\,\mathrm{sh}\,Y, \ P \to -2\exp(-2X) \ (X \to -\infty) \\ &\Psi \to X(\exp N - 1), \ N = Y - X - \ln X, \ N = O(1) \\ &\Psi \to -X\eta, \ \eta = Y/X, \ \eta = O(1), \ P \to -{}^1/_2 \ (X \to \infty) \end{aligned} \tag{3}$$

Transformations (2) enable us to eliminate the arbitrary constants (from the point of view of local consideration) a_0, k, the first of which characterizes the BL profile approaching the separation point, while the second one determines the separation point in the scale of the body /2/. (Compared with the formulation of the problem given in /1/ an additional shift along the Ox' axis is completed here and inaccuracies in these equations are corrected).

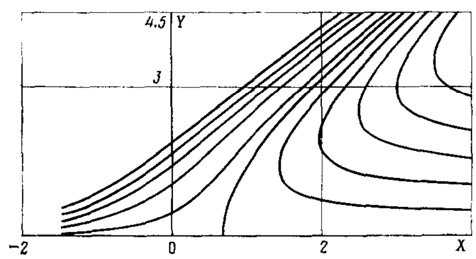

Fig.2

The most important difference between boundary value problem (3) and corresponding problems of separation on a stationary surface (see /4/ is that, as $Y \to \infty$, there is exponential growth of the longitudinal component of the velocity vector $(u = \Psi_Y)$ and not according to a power law. This is due to the fact that, in the case being examined, interaction leads to BL formation, transversely to which, the longitudinal component of the velocity component (ψ_y) varies (close to its minimum line) proportional to the change in the stream function (ψ). The exponential dependence for the pressure $P(X)$ as $X \to -\infty$ follows from the matching with the solution in the interaction region.

Another distinguishing feature of the problem under examination is that the thickness of the displacement at the outer boundary is specified

$$\vartheta = \lim_{Y\to\infty}\left(-\frac{\partial\Psi}{\partial X}\Big/\frac{\partial\Psi}{\partial Y}\right) = 1$$

whereas the pressure distribution remains unknown and, consequently, must be found when solving the problem. (Further details can be found in /1/).

The solutions of boundary value problems for Prandtl's equations for a specified displacement thickness and the usual adhesion conditions at the wall, as was first shown in /5/, have a regular behaviour at the point of zero surface friction and may therefore describe real flows with return streams /6, 7/.

Non-linear boundary problems for flows in which flow separation takes place are usually solved, by numerical methods (see /4/). The solution to problem (3) proves to be surprisingly simple; it is written in the explicit form:

$$\Psi = 2\exp(-X)\,\mathrm{sh}\,Y - Y, \quad P = -{}^1/_2 - 2\exp(-2X) \tag{4}$$

In accordance with the Muir-Rutter-Sears criterion (see /8/) at the separation point the friction and the longitudinal component of the velocity vector simultaneously vanish: $\Psi_Y = \Psi_{YY} = 0$.

From the expression for the stream function (4) the longitudinal coordinate of the separation point $X_s = \ln 2$ and the field of streamlines represented in Fig.2 are found. In this figure the streamlines $\Psi = 0;\ 0.5;\ 1.5;\ 2.5;\ 3.5;\ 4.5$ were constructed in the return flow region $(\Psi < 0)$ and lines were drawn in steps of Ψ of 0.5.

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TRANSPORT EQUATIONS FOR A FIBROUS CONSOLIDATABLE MATERIAL AND THE NEAR-WALL LAYER EFFECT*

N.N. KALININ and B.M. NULLER

The motion of a consolidatable two-phase rod, a fluid-saturated solid elastic porous cylinder, is examined in a cylindrical tube. The effect of the formation and evolution of a near-wall layer is explained qualitatively on the basis of this model. Formulas for the layer thickness and the pore pressure and obtained from the consolidation equations in one limiting case.

Unlike hydraulic transport at low concentrations, the transport of highly-concentrated fibrous materials containing 6-25% solid substance /1-3/ is realized because of the origination of a fluid near-wall layer which reduces the drag tenfold. The theory of this kind of transport has not yet been developed, and existing hydraulic transport models of low-concentration suspensions are not acceptable for this purpose. A highly-concentrated fibrous material is described below by the consolidation equations in the linear approximation.

1. The theory of linear and non-linear consolidation was developed principally in connection with questions of soil mechanics /4-8/. Without taking account of the bulk forces the linear equations of consolidation of a two-phase isotropic porous medium have the form /5/

$$G_1\Delta\mathbf{u} + G_1(1-2\nu_1)^{-1}\,\mathrm{grad}\,\mathrm{div}\,\mathbf{u} + (H_1 - f)\,\mathrm{grad}\,p = 0 \qquad (1.1)$$
$$\partial\theta/\partial t - k\Delta p = 0,\ \theta = H_2\,\mathrm{div}\,\mathbf{u} + (H_3 + H_4)p$$

Here u is the elastic displacement vector, ν_1 is Poisson's ratio, G_1 is the shear modulus, k is the filtration coefficient of the porous medium, p is the fluid pressure in its pores, t is the time, f is the porosity, i.e., the magnitude of the intercommunication pore volume per unit volume of the porous medium, the other closed pores are considered to be part of the solid phase of the skeleton (they substantially diminish the elastic moduli of both the solid phase and the medium as a whole), θ is the change in fluid content per unit volume of the medium, and H_i $(i = 1, 2, 3, 4)$ are the volume strain parameters of the porous medium, its liquid and solid phases.

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